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DIFFERENT DETERIORATION RATES PRODUCTION INVENTORY MODEL WITH TWO WAREHOUSES UNDER SHORTAGES, INFLATION AND PERMISSIBLE DELAY IN PAYMENTS

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Abstract- A two warehouse production inventory model with different deterioration rates under inflation and permissible delay in payments is developed. Demand is considered as function of price and time. Holding cost is also considered as linear function of time. Shortages are allowed and completely backlogged. To represent the model numerical case is considered. Sensitivity analysis is likewise done for parameters.
Keywords – Two warehouse, Different deterioration, Time dependent demand, Price dependent demand, Shortages, Inflation, Permissible Delay in Payments

1. INTRODUCTION

Deteriorating items inventory models were widely studied in past. Ghare and Schrader [8] first developed an EOQ model with constant rate of deterioration. The model was extended by Covert and Philip [7] by considering variable rate of deterioration. Shah [22] further extended the model by considering shortages. The related works are found in (Nahmias [17], Raffat [20], Goyal and Giri [10], Ouyang et al. [18]).

Goyal [9] developed economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [1] extended Goyal's [9] model by considering deterioration. Aggarwal and Jaggi's [1] model was further extended by Jamal et al. [13] to consider shortages. A literature review on inventory model under trade credit is given by Chang et al. [5]. Price sensitive demand under permissible delay in payments inventory model was considered by Teng et al. [23]. An inventory model for exponentially deteriorating items under conditions of permissible delay in payments was developed by Min et al. [16].

In real life situation many time retailers decides to buy goods exceeding their Own Warehouse (OW) capacity to take advantage of price discounts. Therefore an additional stock is arranged as Rented Warehouse (RW) which has better storage facilities with higher inventory holding cost and low rate of deterioration. A two warehouse inventory model was first developed by Hartley [11]. Sarma [21] developed an inventory model with finite rate of replenishment with two warehouses. Other research work related to two warehouse can be found in, for instance (Benkherouf [2], Bhunia and Maiti [3], Kar et al. [14], Chung and Huang [6]). Yang [24] considered a two warehouse inventory problem for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. Bhunia et al. [4] deals with a deterministic inventory model for linear trend in demand under inflationary conditions with different rates of deterioration in two warehouses. Jaggi et al. [12] gave replenishment policy for non-instantaneous deteriorating items in two storage facilities under inflation. A two warehouse inventory models for deteriorating items under conditionally permissible delay in payments was developed by Liang and Zhou [15]. Patel et al. [19] developed a two warehouse production inventory model under shortages, inflation and permissible delay in payments.

For many products it happens that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed two warehouses deteriorating items inventory model.

In this paper we have developed a two warehouse production inventory model with different deterioration rates. Demand function price and time dependent. Shortages are allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

2. ASSUMPTIONS AND NOTATIONS

2.1 Notations:

The following notations are used for the development of the model:

P(t) : Production rate is function of demand at time t, ($\eta D(t)$, $\eta>0$).

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D(t) : Demand is a function of time and price ($a + bt - \rho p$, $a>0$, $0<b<1$, $\rho>0$)
HC(OW) : Holding cost is linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW.
HC(RW) : Holding cost is linear function of time t (x_2+y_2t, $x2>0$, $0<y_2<1$) in RW.
B : Set-up cost per order
c : Purchasing cost per unit
p : Selling price per unit
c_2 : Shortage cost per unit
T : Length of inventory cycle
$I_0(t)$: Inventory level in OW at time t.
$I_r(t)$: Inventory level in RW at time t.
I_e : Interest earned per year
I_p : Interest paid in stocks per year
R : Inflation rate
Q_1 : Inventory level at t_1
Q_2 : Shortage of inventory
Q : Order quantity
t_r : Time at which inventory level becomes zero in RW.
W : Capacity of own warehouse
θ : Deterioration rate in RW and OW during $\mu_1<t<t_1$, $0<\theta<1$
θt : Deterioration rate in RW and OW during $t_1 \le t \le t_0$, $0<\theta<1$
π : Total relevant profit per unit time.

2.2 Assumptions:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- OW has fixed capacity W units and RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory cost per unit in the RW is higher than those in the OW.
- Deteriorated units neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

3. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, production starts at rate η and level of inventory increases to W up to time μ_1 in OW, due to combined effect of production and demand. Then inventory is continued to be stored in RW up to time t_1, production stops at time t_1. During interval (μ_1,t_1) inventory in RW gradually decreases due to demand and deterioration at rate θ, during (μ_1,t_1) inventory in OW depletes due to deterioration at rate θ. During interval (t_1,t_r) inventory in OW depletes due to deterioration at rate θt, inventory in RW depletes due to demand and deterioration at rate θt and reaches to zero at time t_r. During the interval (t_r,t_0) inventory depletes in OW due to demand and deterioration θt. Shortages of size Q_2 units occur during (t_0, t_2). At time t_2 production starts at rate η and inventory starts depleting due to demand and reaches to 0 at time T.

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

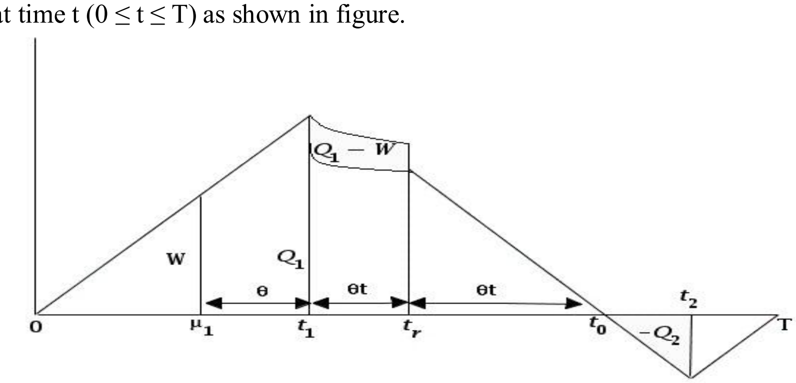


Figure 1 Hence, the inventory level at time t in RW and OW are governed by the following differential equations:

$$\frac{dI_0(t)}{dt} = (\eta\text{-}1)(a + bt - \rho p), \quad 0 \le t \le \mu_1 \tag{1}$$

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = (\eta\text{-}1)(a+bt - \rho p), \quad \mu_1 \le t \le t_1 \tag{2}$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = 0 \quad \mu_1 \le t \le t_1 \tag{3}$$

$$\frac{dI_r(t)}{dt} + \theta t I_r(t) = - (a+bt - \rho p), \quad t_1 \le t \le t_r \tag{4}$$

$$\frac{dI_0(t)}{dt} + \theta t I_0(t) = 0 \quad t_1 \le t \le t_r \tag{5}$$

$$\frac{dI_0(t)}{dt} + \theta t I_0(t) = - (a+bt - \rho p), \qquad t_r \le t \le t_0 \tag{6}$$

$$\frac{dI_0(t)}{dt} = - (a + bt - \rho p), \quad t_0 \le t \le t_2 \tag{7}$$

$$\frac{dI_0(t)}{dt} = (\eta\text{-}1)(a + bt - \rho p), \quad t_2 \le t \le T \tag{8}$$

with initial conditions $I_0(0) = 0$, $I_0(\mu_1) = W$, $I_0(t_1) = W$, $I_0(t_r) = W$, $I_0(t_0) = 0$, $I_0(t_2) = -Q_2$, $I_0(T)=0$, $I_r(0) = 0$, $I_r(\mu_1) = 0$, $I_r(t_1) = Q_1\text{-}W$, and $I_r(t_r) = 0$.
Solving equations (1) to (8) we have,

$$I_0(t) = (\eta\text{-}1)\left[(a - \rho p)t + \frac{1}{2}bt^2\right] \tag{9}$$

$$I_r(t) = (\eta\text{-}1)\left[\begin{array}{l}(a - \rho p)(t - \mu_1) + \frac{1}{2}b\left(t^2\text{-}\mu_1^2\right) + \frac{1}{2}(a - \rho p)\theta\left(t^2\text{-}\mu_1^2\right) \\ +\frac{1}{3}b\theta\left(t^3\text{-}\mu_1^3\right) - (a - \rho p)\theta t(t - \mu_1) - \frac{1}{2}b\theta t\left(t^2\text{-}\mu_1^2\right)\end{array}\right] \tag{10}$$

$$I_0(t) = W\left[1+ \theta(\mu_1 - t)\right] \tag{11}$$

$$I_r(t) = \left[\begin{array}{l}(a - \rho p)(t_r - t) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{1}{6}(a - \rho p)\theta\left(t_r^3 - t^3\right) \\ +\frac{1}{8}b\theta\left(t_r^4 - t^4\right) - \frac{1}{2}(a - \rho p)\theta t^2(t_r - t) - \frac{1}{4}b\theta t^2\left(t_r^2 - t^2\right)\end{array}\right] \tag{12}$$

$$I_0(t) = W\left[1+ \frac{1}{2}\theta(t_1^2 - t^2)\right] \tag{13}$$

$$I_0(t) = \left[\begin{array}{l}(a - \rho p)(t_0 - t) + \frac{1}{2}b\left(t_0^2 - t^2\right) + \frac{1}{6}(a - \rho p)\theta\left(t_0^3 - t^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4 - t^4\right) - \frac{1}{2}(a - \rho p)\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2\left(t_0^2 - t^2\right)\end{array}\right] \tag{14}$$

$$I_0(t) = \left[(a - \rho p)(t_0 - t) + \frac{1}{2}b\left(t_0^2 - t^2\right)\right] \tag{15}$$

$$I_0(t) = (\eta\text{-}1)\left[(a - \rho p)(t - T) + \frac{1}{2}b\left(t^2 - T^2\right)\right] \tag{16}$$

(by neglecting higher powers of θ)
Putting $t = t_1$ in equation (10), we get

$$Q_1 = (\eta\text{-}1)\left[\begin{array}{l}(a - \rho p)(t_1 - \mu_1) + \frac{1}{2}b\left(t_1^2\text{-}\mu_1^2\right) + \frac{1}{2}(a - \rho p)\theta\left(t_1^2\text{-}\mu_1^2\right) \\ +\frac{1}{3}b\theta\left(t_1^3\text{-}\mu_1^3\right) - (a - \rho p)\theta t_1(t_1 - \mu_1) - \frac{1}{2}b\theta t_1\left(t_1^2\text{-}\mu_1^2\right)\end{array}\right] + W \tag{17}$$

Putting $t = t_r$ in equation (13) and (14), we get

$$I_0(t_r) = W\left[1+ \frac{1}{2}\theta(t_1^2 - t_r^2)\right] \tag{18}$$

$$I_0(t_r) = \begin{bmatrix} (a-\rho p)(t_0 - t_r) + \frac{1}{2}b\left(t_0^2 - t_r^2\right) + \frac{1}{6}(a-\rho p)\theta\left(t_0^3 - t_r^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4 - t_r^4\right) - \frac{1}{2}(a-\rho p)\theta t_r^2(t_0 - t_r) - \frac{1}{4}b\theta t_r^2\left(t_0^2 - t_r^2\right) \end{bmatrix} \tag{19}$$

So from equations (18) and (19), we have

$$t_0 = \frac{1}{b\left(-2+\theta tr^2\right)}\left(2a - 2\rho p - a\theta tr^2 + \rho p\theta tr^2 + \sqrt{\begin{pmatrix} 4a^2 - 8a\rho p - 4a^2\theta t_r^2 + 8a\theta t_r^2\rho p + 4\rho^2p^2 - 4\rho^2p^2\theta t_r^2 + \theta^2 t_r^4 a^2 \\ -2\theta^2 t_r^4 a\rho p + \theta^2 t_r^4\rho^2 p^2 - 8b\rho p t_r + 4b^2t_r^2 + 8bW\left(1+\frac{1}{2}\theta t_1^2 - \frac{1}{2}\theta t_r^2\right) \\ +8abt_r + 4b\theta t_r^3\rho p - 2b^2\theta t_r^4 - 4ab\theta t_r^3 - 4b\theta t_r^2 W\left(1+\frac{1}{2}\theta t_1^2 - \frac{1}{2}\theta t_r^2\right) \end{pmatrix}}\right) \tag{20}$$

From equation (20), we see that t_0 is a function of W, t_1 and t_r, so t_0 is not a decision variable.
Similarly putting $t = t_2$ in equations (15) and (16), we get

$$I_0(t_2) = \left[(a-\rho p)(t_0 - t_2) + \frac{1}{2}b\left(t_0^2 - t_2^2\right)\right] \tag{21}$$

$$I_0(t_2) = (\eta - 1)\left[(a-\rho p)(t_2 - T) + \frac{1}{2}b\left(t_2^2 - T^2\right)\right] \tag{22}$$

So from equations (21) and (22), we have

$$t_2 = \frac{1}{\eta b}\left(-\eta a + \eta\rho p + \sqrt{\begin{matrix}\eta^2a^2 - 2\eta^2 a\rho p + \eta^2\rho^2p^2 + 2\eta abt_0 - 2\eta b\rho p t_0 + \eta b^2 t_0^2 - \eta b^2T^2 \\ + 2\eta^2 abT - 2\eta abT - 2\eta^2 b\rho pT + \eta^2 b^2T^2 + 2\eta b\rho pT\end{matrix}}\right) \tag{23}$$

From equation (23), we see that t_2 is a function of t_0 and T so t_2 is not a decision variable.
Putting $t = t_2$ in equation (15), we have

$$Q_2 = \left[(a-\rho p)(t_2 - t_0) + \frac{1}{2}b\left(t_2^2 - t_0^2\right)\right] \tag{24}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Set-up cost (SeC) = B (25)

$$\text{(ii) } HC(OW) = \int_0^{\mu_1}(x_1+y_1t)e^{-Rt}I_0(t)\,dt + \int_{\mu_1}^{t_1}(x_1+y_1t)e^{-Rt}I_0(t)\,dt + \int_{t_1}^{t_r}(x_1+y_1t)e^{-Rt}I_0(t)\,dt + \int_{t_r}^{t_0}(x_1+y_1t)e^{-Rt}I_0(t)\,dt \tag{26}$$

$$\text{(iii) } HC(RW) = \int_{\mu_1}^{t_1}(x_2+y_2t)e^{-Rt}I_r(t)\,dt + \int_{t_1}^{t_r}(x_2+y_2t)e^{-Rt}I_r(t)\,dt \tag{27}$$

$$\text{(iv) } DC = c\left(\int_{\mu_1}^{t_1}\theta I_r(t)\,e^{-Rt}dt + \int_{\mu_1}^{t_1}\theta I_0(t)e^{-Rt}\,dt + \int_{t_1}^{t_r}\theta tI_r(t)e^{-Rt}\,dt + \int_{t_1}^{t_r}\theta tI_0(t)e^{-Rt}\,dt + \int_{t_r}^{t_0}\theta tI_0(t)e^{-Rt}\,dt\right) \tag{28}$$

$$\text{(v) } SC = -c_2\left(\int_{t_0}^{t_2}I_0(t)\,e^{-Rt}dt + \int_{t_2}^{T}I_0(t)e^{-Rt}\,dt\right) \tag{29}$$

$$\text{(vi) } SR = p\left(\int_0^T(a+bt-\rho p)e^{-Rt}dt\right) \tag{30}$$

(by neglecting higher powers of θ)
To determine the interest earned, there will be two cases i.e.
Case I: ($0 \le M \le t_0$) and Case II: ($M > t_0$).
Case I: ($0 \le M \le t_0$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_0.
(vii) Interest earned per cycle:

$$IE_1 = pI_e\int_0^M(a+bt-\rho p)te^{-Rt}dt \tag{31}$$

Case II: ($M > t_0$):
In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So
(viii) Interest earned up to the permissible delay period is:

$$IE_2 = pI_e\left[\int_0^{t_0}(a+bt-\rho p)\,t\,e^{-Rt}dt + (a+bt_0-\rho p)\,t_0\,(M-t_0)\right] \tag{32}$$

To determine the interest payable, there will be five cases i.e.
Interest payable per cycle for the inventory not sold after the due period M is
Case I: ($0\le M\le\mu_1$):

$$\text{(ix)}\quad IP_1 = cI_p\left(\int_M^{\mu_1} I_0(t)e^{-Rt}dt + \int_{\mu_1}^{t_1} I_r(t)e^{-Rt}dt + \int_{\mu_1}^{t_1} I_0(t)e^{-Rt}dt + \int_{t_1}^{t_r} I_r(t)e^{-Rt}dt + \int_{t_1}^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0} I_0(t)e^{-Rt}dt\right) \tag{33}$$

Case II: ($\mu_1\le M\le t_1$):

$$\text{(x)}\quad IP_2 = cI_p\left(\int_M^{t_1} I_r(t)e^{-Rt}dt + \int_M^{t_1} I_0(t)e^{-Rt}dt + \int_{t_1}^{t_r} I_r(t)e^{-Rt}dt + \int_{t_1}^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0} I_0(t)e^{-Rt}dt\right) \tag{34}$$

Case III: ($t_1\le M\le t_r$):

$$\text{(xi)}\quad IP_3 = cI_p\left(\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0} I_0(t)e^{-Rt}dt\right) \tag{35}$$

Case IV: ($t_r\le M\le t_0$):

$$\text{(xii)}\quad IP_4 = cI_p\left(\int_M^{t_0} I_0(t)e^{-Rt}dt\right) \tag{36}$$

Case V: ($M>t_0$):

$$\text{(xiii)}\quad IP_5 = 0 \tag{37}$$

(by neglecting higher powers of b and R)

The total profit (π_i), i=1,2,3,4 and 5 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_i + IE_i\right] \tag{38}$$

Substituting values from equations (25) to (37) in equation (38), we get total profit per unit. Putting $\mu_1 = v_1 t_0$ and value of t_0 and t_2 from equation (20) and (23) in equation (38), we get profit in terms of t_1, tr, T and p for the five cases as under:

$$\pi_1 = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_1 + IE_1\right] \tag{39}$$

$$\pi_2 = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_2 + IE_1\right] \tag{40}$$

$$\pi_3 = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_3 + IE_1\right] \tag{41}$$

$$\pi_4 = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_4 + IE_1\right] \tag{42}$$

$$\pi_5 = \frac{1}{T}\left[SR - SeC - HC(RW) - HC(OW) - DC - SC - IP_5 + IE_2\right] \tag{43}$$

The optimal value of t_1*, tr*, T* and p* (say), which maximizes π_i can be obtained by solving equation (39), (40), (41), (42) and (43) by differentiating it with respect to t_1, tr, T and p and equate it to zero, i.e.

$$\frac{\partial\pi_i(t_1,t_r,T,p)}{\partial t_1}=0,\ \frac{\partial\pi_i(t_1,t_r,T,p)}{\partial t_r}=0,\ \frac{\partial\pi_i(t_1,t_r,T,p)}{\partial t_r}=0,\ \frac{\partial\pi_i(t_1,t_r,T,p)}{\partial t_r}=0,\ i=1,2,3,4,5. \tag{44}$$

provided it satisfies the condition

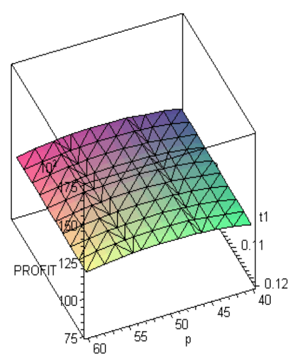
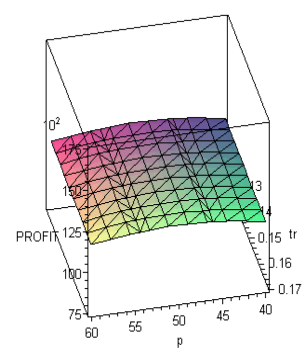
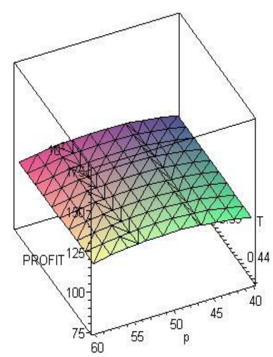
$$\begin{vmatrix} \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_1^2} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_1\partial t_r} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_1\partial T} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_1\partial p} \\ \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_r\partial t_1} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_r^2} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_r\partial T} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial t_r\partial p} \\ \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial T\partial t_1} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial T\partial t_t} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial T^2} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial T\partial p} \\ \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial p\partial t_1} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial p\partial t_r} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial p\partial T} & \frac{\partial\pi_i^2(t_1,t_r,T,p)}{\partial p^2} \end{vmatrix} > 0, \ i=1,2,3,4,5. \tag{45}$$

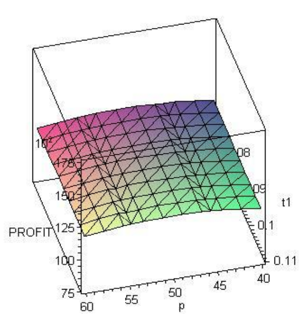
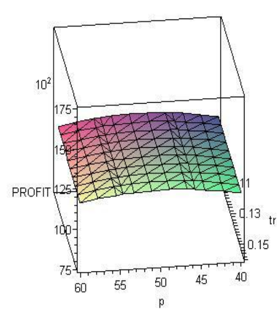
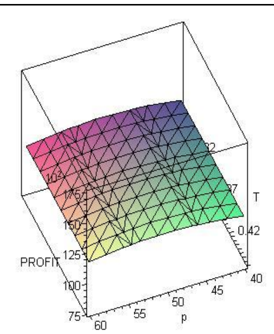
4. NUMERICAL EXAMPLE

Considering B= Rs.100, W = 20, a = 500, b=0.05, c=Rs. 25, ρ= 5, c_2=Rs. 10, θ=0.05, η=2, x_1 = Rs. 3, y_1=0.05, x_2=Rs. 6, y_2=0.06, v_1=0.30, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.02 in appropriate units. The optimal values of t_1, t_r, T, p and Profit for the five cases are shown in table below.

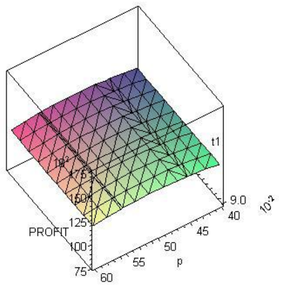
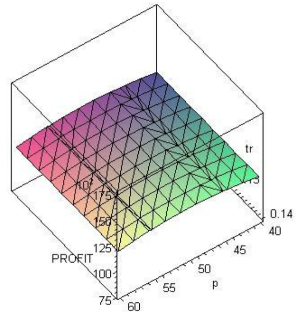
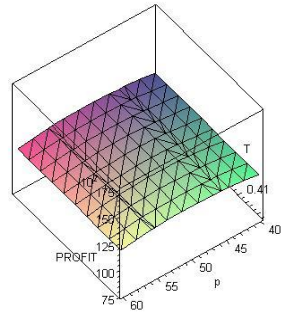
Case	M	t_1	t_r	T	p	Profit
I	0.02	0.1010	0.1467	0.3983	50.2060	11993.4040
II	0.07	0.0936	0.1351	0.3838	50.1903	12007.4780
III	0.11	0.0854	0.1276	0.3665	50.1712	12032.1385
IV	0.16	0.0929	0.1397	0.3546	50.1141	12170.1354
V	0.27	0.1075	0.1629	0.3134	50.1141	12179.4804

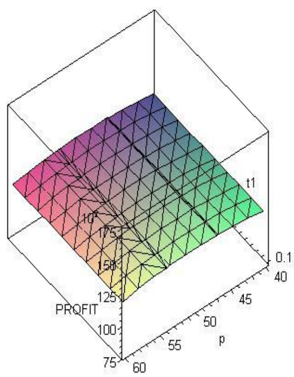
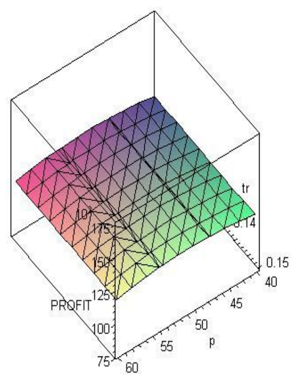
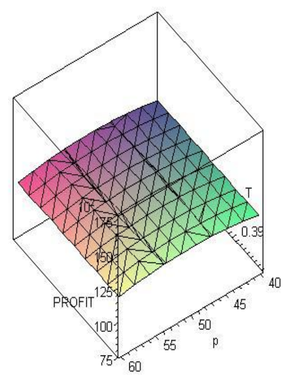
The second order conditions given in equation (37) are also satisfied. The graphical representation of the concavity of the profit function is also given.

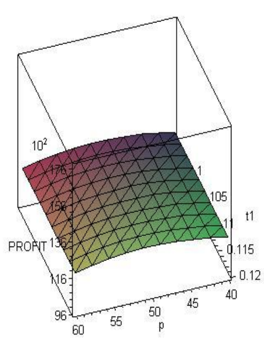
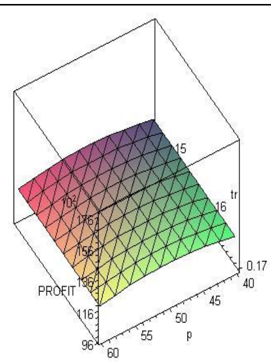
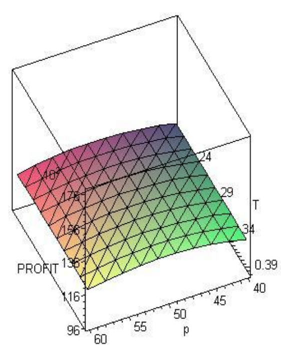
Case I		
t_1, p and Profit	t_r, p and Profit	T, p and Profit
 Graph 1	 Graph 2	 Graph 3

Case II		
t_1, p and Profit	t_r, p and Profit	T, p and Profit
 Graph 4		 Graph 6

	Graph 5	

Case III		
t_1, p and Profit	t_r, p and Profit	T, p and Profit
 Graph 7	 Graph 8	 Graph 9

Case IV		
t_1, p and Profit	t_r, p and Profit	T, p and Profit
 Graph 10	 Graph 11	 Graph 12

Case V		
t_1, p and Profit	t_r, p and Profit	T, p and Profit
 Graph 13	 Graph 14	 Graph 15

5. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1

Case I

Sensitivity Analysis

Para-meter	Change (%)	t_1	t_r	T	p	Profit
θ	+20	0.0888	0.1297	0.3434	60.1764	17414.2800
	+10	0.0945	0.1377	0.3688	55.1900	14578.7293
	-10	0.1085	0.1569	0.4330	45.2249	9658.3479
	-20	0.1170	0.1684	0.4746	40.2477	7573.6239
θ	+20	0.0985	0.1446	0.3973	50.2076	11992.7517
	+10	0.0998	0.1456	0.3978	50.2068	11993.0737
	-10	0.1023	0.1478	0.3988	50.2015	11993.7427
	-20	0.1036	0.1489	0.3992	50.2043	11994.0899
x_1	+20	0.0967	0.1399	0.3963	50.2102	11989.0714
	+10	0.0989	0.1433	0.3973	50.2081	11991.2165
	-10	0.1032	0.1501	0.3993	50.2039	11995.6342
	-20	0.1054	0.1535	0.4002	50.2018	11997.9073
x_2	+20	0.0981	0.1411	0.3954	50.2120	11992.2795
	+10	0.0995	0.1438	0.3968	50.2091	11992.8279
	-10	0.1026	0.1497	0.3999	50.2027	11994.0101
	-20	0.1042	0.1529	0.4015	50.1993	11994.6488
B	+20	0.1161	0.1703	0.4376	50.2242	11945.5500
	+10	0.1087	0.1588	0.4184	50.2153	11968.9150
	-10	0.0924	0.1340	0.3771	50.1963	12019.1958
	-20	0.0843	0.1204	0.3547	50.1860	12046.5230
M	+20	0.1009	0.1464	0.3978	50.2053	11993.9413
	+10	0.1009	0.1466	0.3981	50.2057	11993.6603
	-10	0.1011	0.1468	0.3985	50.2062	11993.1721
	-20	0.1012	0.1469	0.3986	50.2065	11992.9647
R	+20	0.0925	0.1333	0.3761	50.1958	11964.6261
	+10	0.0965	0.1397	0.3867	50.2006	11978.8117
	-10	0.1059	0.1543	0.4109	50.2118	12008.4416
	-20	0.1113	0.1628	0.4248	50.2182	12023.9670
ρ	+20	0.1095	0.1600	0.4205	41.8828	9935.4785
	+10	0.1055	0.1537	0.4100	45.6658	10870.7237
	-10	0.0960	0.1389	0.3853	55.7556	13366.0910
	-20	0.0905	0.1301	0.3707	62.6935	15082.6781
c_2	+20	0.1106	0.1617	0.3930	50.2166	11985.4280
	+10	0.1061	0.1547	0.3954	50.2116	11989.1769
	-10	0.0952	0.1376	0.4016	50.1994	11998.2076
	-20	0.0885	0.1271	0.4054	50.1919	12003.7151

Table 2
Case II
Sensitivity Analysis

Para-meter	Change (%)	t_1	t_r	T	p	Profit
θ	+20	0.0799	0.1157	0.3264	60.1610	17436.6815
	+10	0.0863	0.1249	0.3530	55.1746	14596.6765
	-10	0.1019	0.1466	0.4200	45.2088	9669.0728
	-20	0.1113	0.1545	0.4631	40.2309	7581.4655
θ	+20	0.0910	0.1328	0.3827	50.1921	12006.8925
	+10	0.0923	0.1340	0.3832	50.1912	12007.1810
	-10	0.0950	0.1363	0.3844	50.1894	12007.7837
	-20	0.0963	0.1375	0.3850	50.1885	12008.0984
x_1	+20	0.0887	0.1274	0.3811	50.1952	12003.3315
	+10	0.0911	0.1312	0.3825	50.1927	12005.3803
	-10	0.0961	0.1390	0.3851	50.1880	12009.6249
	-20	0.0986	0.1429	0.3865	50.1856	12011.8212

Para-meter	Change (%)	t_1	t_r	T	p	Profit
x_2	+20	0.0906	0.1293	0.3807	50.1962	12006.5416
	+10	0.0921	0.1321	0.3822	50.1933	12006.9964
	-10	0.0953	0.1383	0.3855	50.1871	12007.9889
	-20	0.0970	0.1417	0.3874	50.1838	12008.5318
B	+20	0.1115	0.1632	0.4266	50.2081	11958.1250
	+10	0.1028	0.1495	0.4058	50.1994	11982.1501
	-10	0.0839	0.1198	0.3605	50.1807	12034.3464
	-20	0.0735	0.1034	0.3356	50.1705	12063.0749
M	+20	0.0887	0.1273	0.3751	50.1863	12014.9356
	+10	0.0912	0.1313	0.3796	50.1885	12011.0202
	-10	0.0959	0.1387	0.3877	50.1917	12004.2974
	-20	0.0981	0.1421	0.3912	50.1926	12001.4679
R	+20	0.0842	0.1202	0.3614	50.1811	11979.7742
	+10	0.0887	0.1271	0.3721	50.1855	11993.4192
	-10	0.0991	0.1437	0.3967	50.1956	12021.9907
	-20	0.1051	0.1531	0.4110	50.2016	12037.0036
ρ	+20	0.1036	0.1509	0.4079	41.8668	9947.2784
	+10	0.0989	0.1434	0.3965	45.6500	10883.5324
	-10	0.0878	0.1259	0.3698	55.7402	13381.7865
	-20	0.0813	0.1157	0.3541	62.6783	15100.5068
c_2	+20	0.1045	0.1521	0.3802	50.1993	11999.5193
	+10	0.0994	0.1442	0.3819	50.1951	12003.2467
	-10	0.0870	0.1247	0.3861	50.1849	12012.3233
	-20	0.0793	0.1126	0.3888	50.1784	12017.9270

Table 3
Case III
Sensitivity Analysis

Para-meter	Change (%)	t_1	t_r	T	p	Profit
θ	+20	0.0773	0.1163	0.3124	60.1378	17473.1886
	+10	0.0811	0.1218	0.3377	55.1533	14626.9132
	-10	0.0900	0.1339	0.4000	45.1924	9688.7973
	-20	0.0951	0.1405	0.4396	40.2179	7596.8515
θ	+20	0.0828	0.1263	0.3660	50.1727	12031.6400
	+10	0.0841	0.1270	0.3662	50.1719	12031.8848
	-10	0.0867	0.1283	0.3668	50.1704	12032.4014
	-20	0.0880	0.1789	0.3670	50.1697	12032.6767
x_1	+20	0.0825	0.1231	0.3659	50.1752	12027.9849
	+10	0.0840	0.1254	0.3662	50.1732	12030.0458
	-10	0.0868	0.1299	0.3668	50.1692	12034.2634
	-20	0.0882	0.1321	0.3671	50.1672	12036.4204
x_2	+20	0.0848	0.1244	0.3649	50.1762	12031.2401
	+10	0.0851	0.1260	0.3657	50.1737	12031.6796
	-10	0.0855	0.1293	0.3674	50.1685	12032.6192
	-20	0.0854	0.1311	0.3682	50.1658	12033.1249
B	+20	0.0962	0.1448	0.4038	50.1938	11980.2148
	+10	0.0909	0.1364	0.3856	50.1827	12005.5481
	-10	0.0795	0.1183	0.3464	50.1592	12060.1931
	-20	0.0733	0.1084	0.3250	50.1466	12089.9823
M	+20	0.0889	0.1332	0.3618	50.1579	12048.0480
	+10	0.0871	0.1304	0.3643	50.1643	12039.9382
	-10	0.0835	0.1247	0.3685	50.1783	12024.6440
	-20	0.816	0.1216	0.3703	50.1858	12017.4501
	+20	0.0801	0.1192	0.3485	50.1604	12005.5088

Parameter	Change (%)	t_1	t_r	T	p	Profit
R	+10	0.0826	0.1233	0.3572	50.1655	12018.6581
	-10	0.0883	0.1323	0.3766	50.1773	12045.9765
	-20	0.0915	0.1374	0.3876	50.1839	12060.2019
ρ	+20	0.1026	0.1551	0.4261	41.8740	9919.1263
	+10	0.0996	0.1502	0.4156	45.6555	10855.7855
	-10	0.0817	0.1219	0.3541	55.7194	13408.8403
	-20	0.0777	0.1154	0.3399	62.6556	15130.5476
c_2	+20	0.0913	0.1371	0.3587	50.1812	12024.5719
	+10	0.0885	0.1326	0.3623	50.1765	12028.1570
	-10	0.0818	0.1220	0.3712	50.1652	12036.5870
	-20	0.0778	0.1155	0.3767	50.1583	12041.5911

Table 4
Case IV
Sensitivity Analysis

Parameter	Change (%)	t_1	t_r	T	p	Profit
θ	+20	0.0833	0.1260	0.2955	60.1117	17530.4774
	+10	0.0880	0.1327	0.3233	55.1260	14673.7424
	-10	0.0982	0.1470	0.3904	45.1634	9719.3673
	-20	0.1040	0.1547	0.4321	40.1881	7621.2050
θ	+20	0.0903	0.1383	0.3541	50.1448	12069.5239
	+10	0.0916	0.1390	0.3543	50.1439	12069.8248
	-10	0.0942	0.1403	0.3549	50.1422	12070.4560
	-20	0.0956	0.1410	0.3551	50.1413	12070.7870
x_1	+20	0.0902	0.1353	0.3542	50.1476	12065.4648
	+10	0.0915	0.1375	0.3544	50.1453	12067.7833
	-10	0.0943	0.1019	0.3547	50.1408	12072.5212
	-20	0.0957	0.1440	0.3549	50.1386	12074.9409
x_2	+20	0.0922	0.1361	0.3530	50.1493	12068.9644
	+10	0.0926	0.1378	0.3538	50.1462	12069.5379
	-10	0.0931	0.1416	0.3555	50.1397	12070.7594
	-20	0.0932	0.1435	0.3563	50.1363	12071.4138
B	+20	0.1041	0.1574	0.3930	50.1639	12016.6319
	+10	0.0986	0.1488	0.3743	50.1536	12042.6970
	-10	0.0869	0.1300	0.3338	50.1322	12099.1886
	-20	0.0804	0.1198	0.3116	50.1210	12130.1786
M	+20	0.0970	0.1462	0.3444	50.1293	12098.0451
	+10	0.0950	0.1430	0.3498	50.1357	12083.7221
	-10	0.0907	0.1360	0.3589	50.1512	12057.2567
	-20	0.0882	0.1322	0.3628	50.1602	12045.0622
R	+20	0.0878	0.1316	0.3372	50.1339	12044.2882
	+10	0.0903	0.1355	0.3456	50.1383	12057.0519
	-10	0.0958	0.1442	0.3643	50.1483	12083.5642
	-20	0.0989	0.1491	0.3749	50.1540	12097.3673
ρ	+20	0.0998	0.1506	0.3783	41.8223	10000.9532
	+10	0.0965	0.1454	0.3672	45.6042	10941.2359
	-10	0.0888	0.1331	0.3403	55.6912	13450.6920
	-20	0.0840	0.1255	0.3238	62.6273	15177.5476
c_2	+20	0.0978	0.1474	0.3474	50.1500	13064.5070
	+10	0.0955	0.1438	0.3508	50.1468	12047.1699
	-10	0.0900	0.1350	0.3589	50.1388	12073.4588
	-20	0.0867	0.1297	0.3639	50.1338	12077.0100

Table 5
Case V
Sensitivity Analysis

Para-meter	Change (%)	t_1	t_r	T	p	Profit
θ	+20	0.1007	0.1538	0.2416	60.1017	17708.7717
	+10	0.1039	0.1580	0.2761	55.1052	14814.0894
	-10	0.1121	0.1690	0.3550	45.1276	9803.4495
	-20	0.1179	0.1768	0.4028	40.1458	9684.9037
θ	+20	0.1050	0.1618	0.3132	50.1163	12178.5615
	+10	0.1063	0.1624	0.3133	50.1152	12179.0151
	-10	0.1088	0.1634	0.3135	50.1129	12179.9575
	-20	0.1101	0.1640	0.3136	50.1118	12180.4469
x_1	+20	0.1056	0.1598	0.3146	50.1193	12173.3449
	+10	0.1066	0.1614	0.3140	50.1167	12176.3957
	-10	0.1085	0.1644	0.3128	50.1114	12182.5995
	-20	0.1094	0.1660	0.3121	50.1088	12185.7533
x_2	+20	0.1071	0.1597	0.3124	50.1237	12177.5200
	+10	0.1074	0.1613	0.3129	50.1189	12178.4853
	-10	0.1075	0.1645	0.3139	50.1090	12180.5081
	-20	0.1073	0.1663	0.3144	50.1039	12181.5728
B	+20	0.1169	0.1778	0.3532	50.1261	12119.4814
	+10	0.1124	0.1706	0.3339	50.1199	12148.5857
	-10	0.1023	0.1547	0.2915	50.1087	12212.5413
	-20	0.0968	0.1458	0.2678	50.1041	12248.2979
M	+20	0.1174	0.1785	0.2889	50.1185	12246.9676
	+10	0.1126	0.1710	0.3019	50.1146	12212.0138
	-10	0.1021	0.1543	0.3235	50.1161	12149.1106
	-20	0.0963	0.1451	0.3322	50.1204	12120.7086
R	+20	0.1039	0.1571	0.2989	50.1103	12156.3505
	+10	0.1057	0.1599	0.3059	50.1121	12167.7801
	-10	0.1095	0.1661	0.3215	50.1163	12191.4714
	-20	0.1117	0.1696	0.3303	50.1188	12203.7760
ρ	+20	0.1146	0.1741	0.3458	41.7901	10089.3902
	+10	0.1111	0.1686	0.3305	45.5733	11038.8670
	-10	0.1039	0.1570	0.2938	55.6652	13575.3822
	-20	0.1001	0.1510	0.2709	62.6065	15323.0927
c_2	+20	0.1093	0.1657	0.3082	50.1169	12177.7491
	+10	0.1085	0.1643	0.3107	50.1156	12178.5667
	-10	0.1065	0.1613	0.3165	50.1123	12180.5084
	-20	0.1053	0.1594	0.3201	50.1102	12181.6739

From the table we observe that as parameter a increases/ decreases average total profit increases/ decreases for all five cases.
From the table we observe that as parameter θ and x_2 increases/ decreases there is very minor decrease/increase in average total profit for all five cases.
From the table we observe that as parameters x_1, B, R and ρ increases/ decreases average total profit decreases/ increases for all five cases.
From the table we observe that as parameter M increases/ decreases there is very minor increase/decrease in average total profit for case I and there is increases/ decreases in average total profit for remaining four cases.
From the table we observe that as parameter c_2 increases/ decreases there is decreases/ increases in average total profit for first four cases and there is very minor decrease/increase in average total profit for case V.

6. CONCLUSION

A two warehouse production inventory model for deteriorating items with different deterioration rates under price and time dependent demand is developed in this paper. Shortages are considered and holding cost is time varying. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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